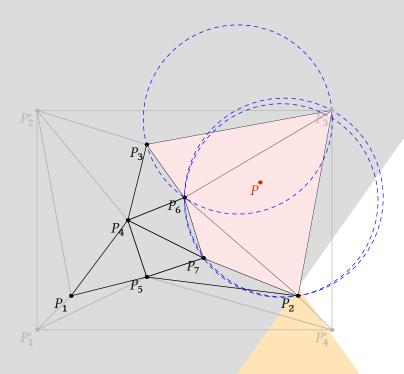
luamesh

compute and draw meshes with LuaFTEX



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http://melusine.eu.org/syracuse/G/delaunay/

luamesh: compute and draw meshes with Lua&TEX

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The package luamesh allows to compute and draw 2D Delaunay triangulation. The algorithm is written with lua, and depending on the choice of the "engine", the drawing is done by MetaPost (with luamplib) or by tikz.

The Delaunay triangulation algorithm is the Bowyer and Watson algorithm. Several macros are provided to draw the global mesh, the set of points, or a particular step of the algorithm.

I would like to thank Jean-Michel Sarlat, who hosts the development with a git project on the melusine machine:

https://melusine.eu.org/syracuse/G/delaunay/

I would also like to thank the first user, an intensive test user, and a very kind English corrector: Nicole Spillane.

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1 Installation

Of course, you can just put the two files luamesh.lua and luamesh.sty in the working directory, but this is not recommended.

1.1 With TEXlive and Linux or Mac OSX

To install Luamesh with TeXlive, you have to create the local texmf directory in your home.

```
user $> mkdir ~/texmf
```

Then place the files in the correct directories. First, the luamesh.sty file must be in the directory:

~/texmf/tex/latex/luamesh/

and secondly, the luamesh.lua must be in the directory:

~/texmf/scripts/luamesh/

Once you have done this, luamesh can be included in your document with

\usepackage{luamesh}

1.2 With MikTEX and Windows

We do not know these two systems, so we refer to the documentation for integrating local additions to MikTeX:

http://docs.miktex.org/manual/localadditions.html

1.3 A LuaLTFX package

If you want to use this package, you must compile your document with lualatex:

user \$> lualatex mylatexfile.tex

1.4 Dependencies

This package is built upon two main existing packages to draw the triangulations:

- 1. luamplib to use MetaPost via the LuaTeX library mplib;
- 2. and tikz.

We will see how to choose between these two drawing engines.

Moreover, the following packages are necessary:

- 1. xkeyval to manage the optional arguments;
- 2. xcolor to use colors (needed by luamplib);
- 3. ifthen to help the programming with TEX.

2 The Basic Macros

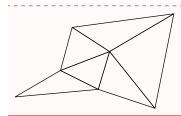
Let us recall that this package provides macros to draw two dimensional triangulations (or meshes).

2.1 Draw a Complete Mesh

\buildMeshBW[\langle options \rangle] \{ \langle list of points \rangle or \langle file name \rangle \}

This macro produces the Delaunay triangulation (using the Bowyer and Watson algorithm) of the given $\langle list\ of\ points \rangle$. The list of points must be given in the following way :

\buildMeshBW{(0.3,0.3);(1.5,1);(4,0);(4.5,2.5);(1.81,2.14);(2.5,0.5);(2.8,1.5)}



2.1.1 The Options

There are several options to customize the drawing.

mode = int (default) or ext: this option allows to use either the previously described set
 of points in the argument, or a file, containing, line by line (2 columns), the points. Such
 a file looks like:

x1 y1

x2 y2

x3 y3

. . .

xn yn

bbox = none (default) or show: this option allows to draw the added points to form a bounding box^1 and the corresponding triangulation. By default, these triangles are removed at the end of the algorithm.

color = \(value \rangle \) (default: black): The color of the drawing.

print = none (default) or points: To label the vertices of the triangulation. This also
 adds a dot at each vertex.

meshpoint = \(value \) (default: P): The letter(s) used to label the vertices of the triangulation. It is included in the math mode delimiters \$...\$. The bounding box points are labeled with numbers 1 to 4 and with a star exponent.

tikz (boolean, default:false): By default, this boolean is set to false, and MetaPost (with luamplib) is used to draw the picture. With this option, tikz becomes the *drawing engine*.

scale = \(value \) (default: 1cm): The scale option defines the scale at which the picture is
drawn (the same for both axes). It must contain the unit of length (cm, pt, etc.).

To illustrate the options, let us show you an example. We consider a file mesh.txt:

¹The bounding box is defined by four points place at 15% around the box defined by (x_{\min}, y_{\min}) , (x_{\min}, y_{\max}) , (x_{\max}, y_{\max}) , and (x_{\min}, y_{\max}) . It is used by the algorithm and will be computed in any case.

```
0.3 0.3
1.5 1
4 0
4.5 2.5
1.81 2.14
2.5 0.5
2.8 1.5
```

```
\buildMeshBW[%
tikz,
mode = ext,
bbox = show,
color = red,
colorBbox = blue!30,
print = points,
meshpoint = x,
scale = 1.3cm,
]{mesh.txt}
```

The drawing engine is not very relevant here, but it is useful to understand how the drawing is made. However, the engine will be relevant to the so called *inc* macros (section 3), for adding code before and after the one generated by luamesh.

2.2 Draw the Set of Points

\drawPointsMesh[\langle options \rangle] \{ \langle list of points \rangle or \langle file name \rangle \}

With the \drawPointsMesh, we plot the set of (user chosen) points from which the Bowyer and Watson algorithm computes the triangulation.

The use of this macro is quite similar to \buildMeshBW. Here is an example of the basic uses.

```
\drawPointsMesh{(0.3,0.3);(1.5,1);(4,0);(4.5,2.5);(1.81,2.14);(2.5,0.5);(2.8,1.5)}

•
•
•
•
•
•
•
•
•
```

2.2.1 The Options

There are several options (exactly the same as for the \buildMeshBW) to customize the drawing.

mode = int (default) or ext: this option allows to use either the previously described set
 of points as the argument, or a file, containing, line by line (2 columns), the points. Such
 a file looks like:

```
x1 y1 x2 y2
```

x2 y2

... xn yn

bbox = none (default) or show: this option allows to draw the added points to form a bounding box and the corresponding triangulation. By default, these triangles are removed at the end of the algorithm. Here, because we plot only the vertices of the mesh, there are no triangles, only dots.

```
color = \langle value \rangle (default: black): The color of the drawing.
```

- print = none (default) or points: To label the vertices of the triangulation. This also
 adds a dot at each vertex. Without label, there is still the dot.
- meshpoint = \(\value \rightarrow \) (default: P): The letter(s) used to label the vertices of the triangulation. It is included in the math mode delimiters \$...\$. The bounding box points are labeled with numbers 1 to 4 and with a star exponent.
- tikz (boolean, default:false): By default, this boolean is set to false, and MetaPost (with luamplib) is used to draw the picture. With this option, tikz becomes the drawing engine.
- scale = \langle value \rangle (default: 1cm): The scale option defines the scale at which the picture is
 drawn (the same for both axes). It must contain the unit of length (cm, pt, etc.).

With the same external mesh point file presented in section 2.1, we illustrate the different options.

```
\drawPointsMesh[%
 tikz,
 mode = ext,
 bbox = show,
 color = blue,
 colorBbox = red,
 print = points,
 meshpoint = y,
 scale = 1.3cm,
 ]{mesh.txt}
y_2^*
                                                          y_3^*
                       y_5
                               y<sub>6</sub>
       y_1
                                               y<sub>3</sub>
y_1^*
                                                           y_4^*
```

2.3 Draw a Step of the Bowyer and Watson Algorithm

 $\mbox{\mbox{$\$

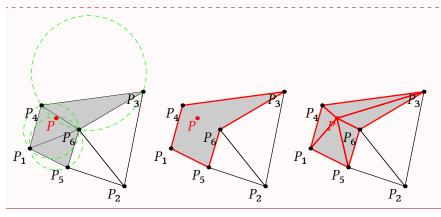
This command allows to plot the steps within the addition of a point in a Delaunay triangulation, by the Bowyer and Watson algorithm.

This macro produces the Delaunay triangulation (using the Bowyer and Watson algorithm) of the given $\langle \textit{list of points} \rangle$ and shows a step of the algorithm when the $\langle \textit{point} \rangle$ is added. The list of points must be given in the following way:

and the point is of the form (x,y). The $\langle file\ name \rangle$ and $\langle number\ of\ line \rangle$ will be explained in the option description.

One can use the macro as fallows:

 $\label{lem:meshAddPointBW[step=newtriangles]} $$ \operatorname{(1.5,1);(4,0);(4.5,2.5);(1.81,2.14);(2.5,0.5);(2.8,1.5) } $$ $$ \{(2.2,1.8)\}$$$



The default value for step is badtriangles. Consequently, the first line is equivalent to

```
\meshAddPointBW{(1.5,1);(4,0);(4.5,2.5);(1.81,2.14);(2.5,0.5);(2.8,1.5)}{(2.2,1.8)}
```

2.3.1 The Options

There are several options (some of them are the same as for \buildMeshBW) to customize the drawing.

mode = int (default) or ext: this option allows to use either the previously described set
 of point in the first argument, or a file containing, line by line (2 columns), the points.
 Such a file looks like:

x1 y1

x2 y2

x3 y3

. . .

xn yn

For the second argument of the macro, if we are in the mode = ext, the argument must be the *line number* of the file corresponding to the point we want to add. The algorithm will stop the line before to build the initial triangulation for which it will add the point corresponding to the line. The subsequent lines in the file are ignored.

bbox = none (default) or show: this option allows to draw the added points to form a bounding box and the corresponding triangulation. By default, these triangles are removed at the end of the algorithm.

color = \langle value \rangle (default: black): The color of the drawing.

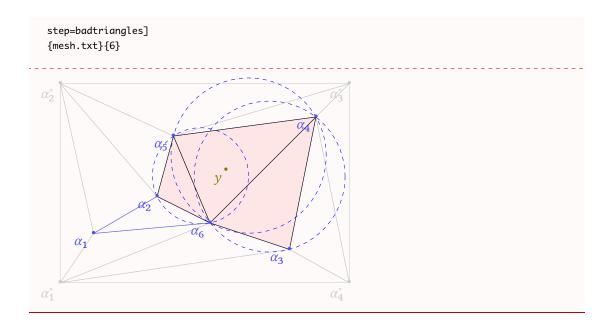
- colorNew = \(\nabla value \rangle\) (default: red): The color of the drawing of the "new" elements which are the point to add, the polygon of the cavity, and the new triangles.
- colorBack = \langle value \rangle (default: black!20): The color for the filling of the region concerned
 by the addition of the new point.
- colorCircle = \langle value \rangle (default: green): The color for the circumcircle of the triangles
 containing the point to add.
- meshpoint = \(value \) (default: P): The letter(s) used to label the vertices of the triangulation. It is included in the math mode delimiters \$...\$. The bounding box points are labeled with numbers 1 to 4 and with a star exponent.
- step = badtriangles (default) or cavity or newtriangles: To choose the step we want
 to draw, corresponding to the steps of the Bowyer and Watson algorithm.
- newpoint = \langle value \rangle (default: P): The letter(s) used to label the new point of the triangulation. It is include in the math mode delimiters \$...\$.
- tikz (boolean, default:false): By default, this boolean is set to false, and MetaPost (with luamplib) is used to draw the picture. With this option, tikz is the *drawing engine*.
- scale = \(value \) (default: 1cm): The scale option defines the scale at which the picture is
 draw (the same for the two axis). It must contain the unit of length (cm, pt, etc.).

Here is an example of customizing the drawing. First, recall that the external file mesh.txt is:

- 0.3 0.3
- 1.5 1
- 4 0
- 4.5 2.5
- 1.81 2.14
- 2.5 0.5
- 2.8 1.5

We draw the addition of the 6th point. The 7th line will be ignored.

```
\meshAddPointBW[
tikz,
mode = ext,
color = blue!70,
meshpoint = \alpha,
newpoint = y,
colorBack=red!10,
colorNew = green!50!red,
colorCircle = blue,
colorBbox = black!20,
bbox = show,
scale=1.4cm,
```



3 The inc Macros

The three macros presented in the above sections have complementary macros, with the suffix inc that allow the user to add code (MetaPost or tikz, depending of the drawing engine) before and after the code generated by luamesh.

The three macros are:

```
\buildMeshBWinc[\langle options\rangle] \{\langle list of points \rangle or \langle file name\rangle \} \\ \langle code before\rangle \]
```

3.1 With MetaPost

We consider the case where the drawing engine is MetaPost (through the <code>luamplib</code> package). We describe the feature taking one macro in example but the mechanism and the possibilities are exactly the same for all the macros.

When we use the MetaPost drawing engine, the macros previously described produced a code of the form

```
\begin{luamplib}
u:=<scale>;
beginfig(0);
<code for the drawing>
endfig;
\end{luamplib}
```

Then, the arguments *⟨code before⟩* and *⟨code after⟩* are inserted as follows:

With the *inc* macros, the user has to add the beginfig(); and endfig; commands to produce a picture. Indeed, this allows to use the \everymplib command from the \luamplib package.

3.1.1 The LaTEX Colors Inside the MetaPost Code

The configurable colors of the LATEX macro are accessible inside the MetaPost code. For \buildMeshBWinc and \drawPointsMeshinc, we have \luameshmpcolor, and \luameshmpcolorBbox. For the macro \meshAddPointBWinc we have three additional colors: \luameshmpcolorBack, \luameshmpcolorNew, and \luameshmpcolorCircle. Of course, we can define MetoPost colors as well. Finally, the luamplib mechanism of \mpcolor is also available.

3.1.2 The Mesh Points

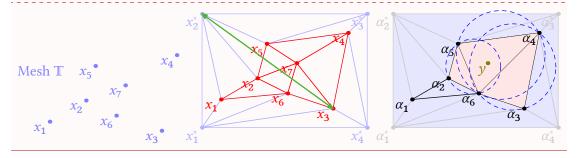
At the beginning of the automatically generated code, a list of MetaPost pairs are defined corresponding to all the vertices of the mesh (when the option bbox=show, the last 4 points are the bounding box points). The points are available with the MeshPoints[] table of variables. The MeshPoints[i] are defined using the unit length u.

3.1.3 Examples

Here is three examples for the different macros.

```
\drawPointsMeshinc[
color = blue!50,
print = points,
meshpoint = x,
scale=0.8cm,
]{(0.3,0.3);(1.5,1);(4,0);(4.5,2.5);(1.81,2.14);(2.5,0.5);(2.8,1.5)}%
{% code before
   beginfig(0);
}%
{% code after
   label(btex Mesh $\mathbb{T}$$ etex, (0,2u)) withcolor \luameshmpcolor;
endfig;
}
\buildMeshBWinc[%
bbox = show,
```

```
color = red,
colorBbox = blue!30,
print = points,
meshpoint = x,
scale=0.8cm
]{(0.3,0.3);(1.5,1);(4,0);(4.5,2.5);(1.81,2.14);(2.5,0.5);(2.8,1.5)}%
{% code before
 beginfig(0);
{% code after
 drawdblarrow MeshPoints[3] -- MeshPoints[9] withpen pencircle scaled 1pt
 withcolor (0.3,0.7,0.2);
 endfig;
}
\meshAddPointBWinc[
meshpoint = \alpha,
newpoint = y,
colorBack=red!10,
colorNew = green!50!red,
colorCircle = blue,
colorBbox = black!20,
bbox = show,
scale=0.8cm,
step=badtriangles]
\{(0.3,0.3);(1.5,1);(4,0);(4.5,2.5);(1.81,2.14);(2.5,0.5)\}\{(2.8,1.5)\}\%
{%code before
 picture drawing;
 drawing := image(
}{%code after
 );
 beginfig(0);
 fill MeshPoints[7]--MeshPoints[8]--MeshPoints[9]--MeshPoints[10]--cycle
 withcolor \mpcolor{blue!10};
 draw drawing;
 endfig;
}
```



The variables MeshPoints[] are not defined for the argument corresponding to the code to place before the code generated by luamesh. Hence, to use such variables, we have to define a picture as shown in the third example above.

3.2 With TikZ

If we have chosen tikz as the engine drawing, the added code will be written in tikz. In that case, the two arguments ⟨code before⟩ and ⟨code after⟩ will be inserted as follows:

Because the engine is tikz their is no issue with colors, the LaTeX colors (e.g.: xcolor) can be directly used.

3.2.1 The Mesh Points

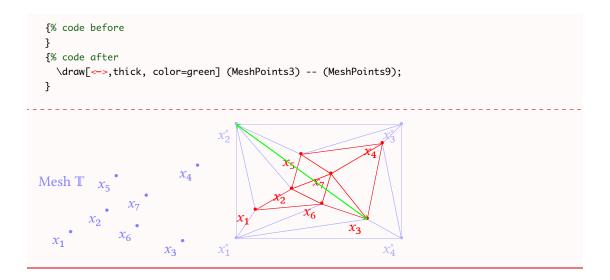
The points of the mesh are defined here as tikz \coordinate named as follows

```
\coordinate (MeshPoints1) at (...,...);
\coordinate (MeshPoints2) at (...,...);
\coordinate (MeshPoints3) at (...,...);
%etc.
```

Once again these coordinates are not yet defined for the *<code before>* argument.

3.2.2 Examples

```
\drawPointsMeshinc[
tikz,
color = blue!50,
print = points,
meshpoint = x,
scale=0.8cm,
]{(0.3,0.3);(1.5,1);(4,0);(4.5,2.5);(1.81,2.14);(2.5,0.5);(2.8,1.5)}%
{% code before
}%
{% code after
  \label{eq:color} $$ \ode[color = blue!50] $ at (0,2) {Mesh $\mathbb{T}}; $$ ;
\buildMeshBWinc[%
tikz,
bbox = show,
color = red,
colorBbox = blue!30,
print = points,
meshpoint = x,
scale=0.8cm
]{(0.3,0.3);(1.5,1);(4,0);(4.5,2.5);(1.81,2.14);(2.5,0.5);(2.8,1.5)}%
```

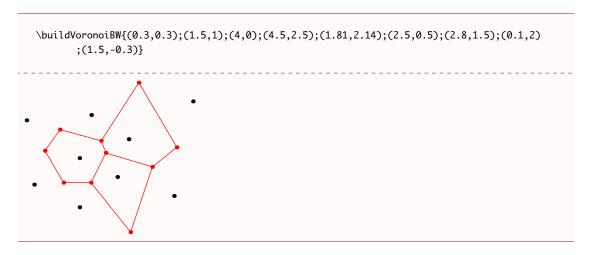


4 Voronio Diagrams

Another interesting concept of Delaunay triangulation is that it is *dual* to it so-called Voronio diagram. For a finite set of points $\{p_1, \dots, p_n\}$ in the Euclidean plane, for all p_k , it corresponds a Voronoi cell R_k consisting of every point in the Euclidean plane whose distance to p_k is less than or equal to its distance to any other $p_{k'}$.

\buildVoronoiBW[\langle options \rangle] \{ \langle list of points \rangle or \langle file name \rangle \}

This macro produce the Voronio diagram dual to the Delaunay triangulation (computed by the Bowyer and Watson algorithm) of the given $\langle \textit{list of points} \rangle$. Once again, the list of points must be given in the following way :



4.1 The Options

There are several options to customize the drawing.

- mode = int (default) or ext: this option allows to use either the previously described set
 of points in the argument, or a file, containing, line by line (2 columns), the points. Such
 a file looks like:
 - x1 y1
 - x2 y2
 - x3 y3
 - . . .
 - xn yn
- bbox = none (default) or show: this option allows to draw the added points to form a bounding box^2 and the corresponding triangulation. By default, these points are removed at the end of the algorithm.
- color = \(\nabla value \rangle\) (default: black): The color of the drawing.
- colorVoronoi = \(\nabla alue \rangle \) (default: black): The color of the drawing for the elements (points and polygons) belonging to the Voronoi diagram.
- print = none (default) or points: To label the vertices of the triangulation. Contrary to
 the previous macros, where print=none, a dot is produced at each vertex (of the set of
 points and on the circumcircle center which are the nodes of the Voronoi diagram).
- meshpoint = \(\nabla value \rangle\) (default: P): The letter(s) used to label the vertices of the triangulation. It is included in the math mode delimiters \$...\$. The bounding box points are labeled with numbers 1 to 4 and with a star exponent.
- circumpoint = \langle value \rangle (default: P): The letter(s) used to label the vertices of the Voronoi
 diagram. It is included in the math mode delimiters \$...\$.
- tikz (boolean, default:false): By default, this boolean is set to false, and MetaPost (with luamplib) is used to draw the picture. With this option, tikz becomes the drawing engine.
- scale = \(\nabla value \rangle\) (default: 1cm): The scale option defines the scale at which the picture is drawn (the same for both axes). It must contain the unit of length (cm, pt, etc.).

²The bounding box is defined by four points place at 15% around the box defined by (x_{\min}, y_{\min}) , (x_{\min}, y_{\max}) , (x_{\max}, y_{\max}) , and (x_{\min}, y_{\max}) . It is used by the algorithm and will be computed in any case.

4.2 The inc variant

Once again, a variant of the macros is available allowing the user to add code before and after the code produced by luamesh. We refer to the section 3 because it works the same way.

Let us note that:

- with MetaPost, the circumcenters are defined using pair CircumPoints[];, and so, are accessible.
- With tikz, there are new coordinates defined as follows

```
\coordinate (CircumPoints1) at (...,..);
\coordinate (CircumPoints2) at (...,..);
\coordinate (CircumPoints3) at (...,..);
% etc.
```

Finally, when the MetaPost drawing engine is used, another color is available (see 3.1.1): \luameshmpcolorVoronoi.

5 With Gmsh

Gmsh is a open source efficient software that produces meshes. The exporting format is the MSH ASCII file format and can be easily read by a Lua program. Luamesh provides the user with dedicated macros to read and draw meshes coming from a Gmsh exportation.

```
\drawGmsh[\langle options \rangle] \{\langle file name \rangle \}
```

This macro draw the triangulation produced by Gmsh and exported in the msh format. The argument is the name of the file to read (e.g.: maillage.msh).

```
\drawGmsh{maillage.msh}
```

There are several options to customize the drawing.

```
color = \langle value \rangle (default: black): The color of the drawing.
```

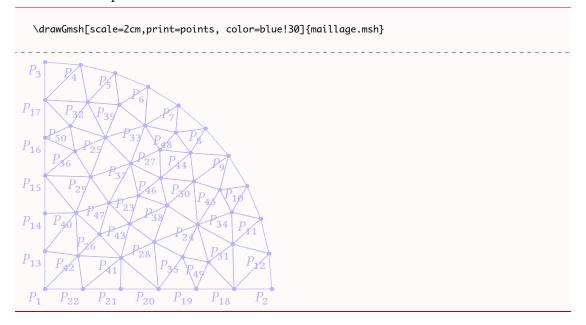
print = none (default) or points: To label the vertices of the triangulation. Contrary to
 the previous macros, where print=none, a dot is produced at each vertex (of the set of
 points and on the circumcircle center which are the nodes of the Voronoi diagram).

meshpoint = \(value \) (default: P): The letter(s) used to label the vertices of the triangulation. It is included in the math mode delimiters \$...\$. The bounding box points are labeled with numbers 1 to 4 and with a star exponent.

tikz (boolean, default:false): By default, this boolean is set to false, and MetaPost (with luamplib) is used to draw the picture. With this option, tikz becomes the drawing engine.

scale = \(value \) (default: 1cm): The scale option defines the scale at which the picture is
drawn (the same for both axes). It must contain the unit of length (cm, pt, etc.).

Here is an example:



5.1 Gmsh and Voronoi Diagrams

Because Gmsh generates Delaunay triangulations, we can plot the Voronoi diagram associated. This is done by the following macro:

\gmshVoronoi[\langle options\rangle] \{\langle file name\rangle\}

\gmshVoronoi{maillage.msh}



We refer to the section 4.1 for the list of the options.

5.2 The *inc* variants

Once again, there exists inc variant macros:

We refer to the previous sections for explanations.

6 Gallery

6.1 With Animate

If you use *adobe acrobat reader*, you can easily produce an animation of the Bowyer and Watson algorithm with the package animate.

For example, the following code (in a file name animation.tex):

```
\documentclass{article}
%% lualatex compilation
\usepackage[margin=2.5cm]{geometry}
\usepackage{luamesh}
\usepackage{fontspec}
\usepackage{multido}
\pagestyle{empty}
\def\drawPath{draw (-2,-2)*u--(8,-2)*u--(8,6)*u--(-2,6)*u--cycle} \ with color 0.99 white;}
\def\clipPath{clip} currentpicture to (-2,-2)*u-(8,-2)*u-(8,6)*u-(-2,6)*u--cycle;}
\begin{document}
\drawPointsMeshinc[
mode=ext,
bbox = show,
colorBbox = blue!20,
print=points
{mesh.txt}%
{%
  beginfig(0);
```

```
\drawPath
}%
{%
  \clipPath
  endfig;
\newpage
\buildMeshBWinc[
mode=ext,
bbox = show,
colorBbox = blue!20,
print=points
]
{meshInit.txt}%
{%
 beginfig(0);
  \drawPath
}%
  \clipPath
  endfig;
\mbox{multido}{ii=5+1}{4}{\%}
  \newpage
  \meshAddPointBWinc[
  mode=ext,step=badtriangles,
  colorNew =green!20!red,
  colorBack=red!10,
  colorCircle = blue,
  bbox = show,
  colorBbox = blue!20
  {mesh.txt}{ii}%
  {%
    beginfig(0);
    \drawPath
  }%
  {%
    \clipPath
    endfig;
  } \newpage
  \meshAddPointBWinc[
  mode=ext,step=cavity,
  colorNew =green!20!red,
  colorBack=red!10,
  colorCircle = blue,
  bbox = show,
  colorBbox = blue!20
  {mesh.txt}{ii}%
    beginfig(0);
    \drawPath
```

```
}%
    {%
      \clipPath
      endfig;
    } \newpage
    \meshAddPointBWinc[
    mode=ext,step=newtriangles,
    colorNew =green!20!red,
    colorBack=red!10,
    colorCircle = blue,
    bbox = show,
    colorBbox = blue!20]
    \{\mathsf{mesh.txt}\}\{\setminus ii\}\%
      beginfig(0);
      \drawPath
    }%
    {%
      \clipPath
      endfig;
    }
  }
  \newpage
  \buildMeshBWinc[
  mode=ext,
  bbox = show,
  colorBbox = blue!20,
  print=points
  ]
  {mesh.txt}%
  {%
    beginfig(0);
    \drawPath
 }%
  {%
    \clipPath
    endfig;
  }
  \newpage
  \buildMeshBWinc[
  mode=ext,
  print=points
  ]
  {mesh.txt}%
  {%
    beginfig(0);
    \drawPath
 }%
  {%
    \clipPath
    endfig;
\end{document}
```

produces a PDF with multiple pages. Using the pdfcrop program, we crop the pages to the material, and then we can animate the PDF using the animate package.

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