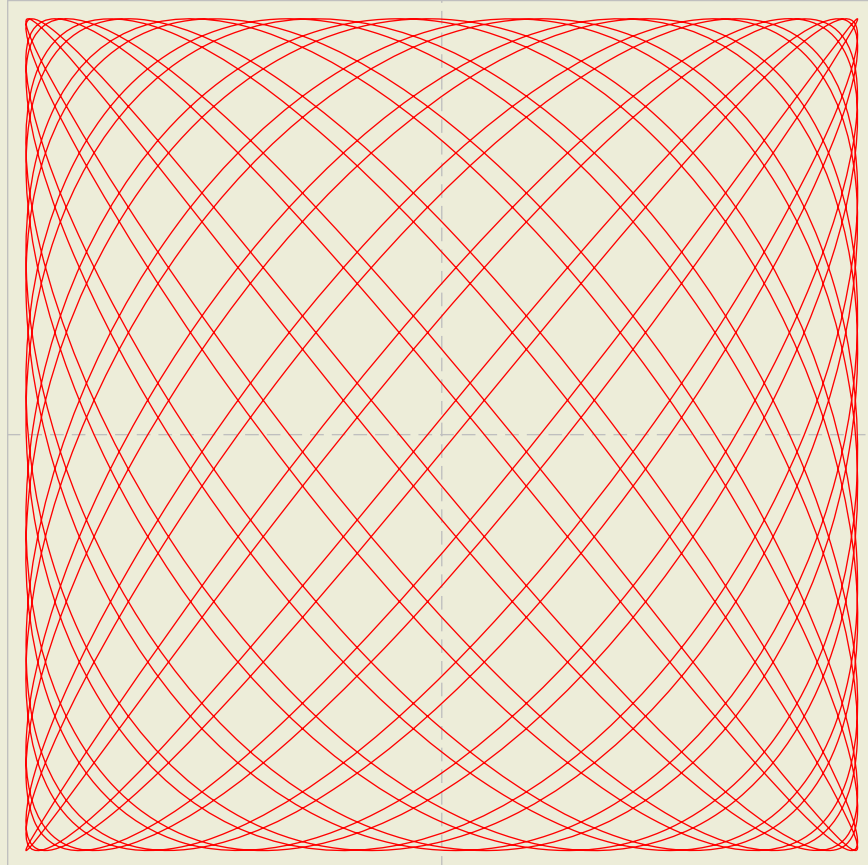


Some PSTricks macros for the study of oscillations



LISSAJOUS *curves*

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1 LISSAJOUS curves: harmonic oscillations in two dimensions

The direction of the oscillations is perpendicular – the directions refer to the x - and y -direction of a Cartesian Coordinate System. We describe the two-dimensional orbit of a material point, that is driven by two harmonic oscillations. The orbit is determined by the superposition of the two harmonic oscillations.

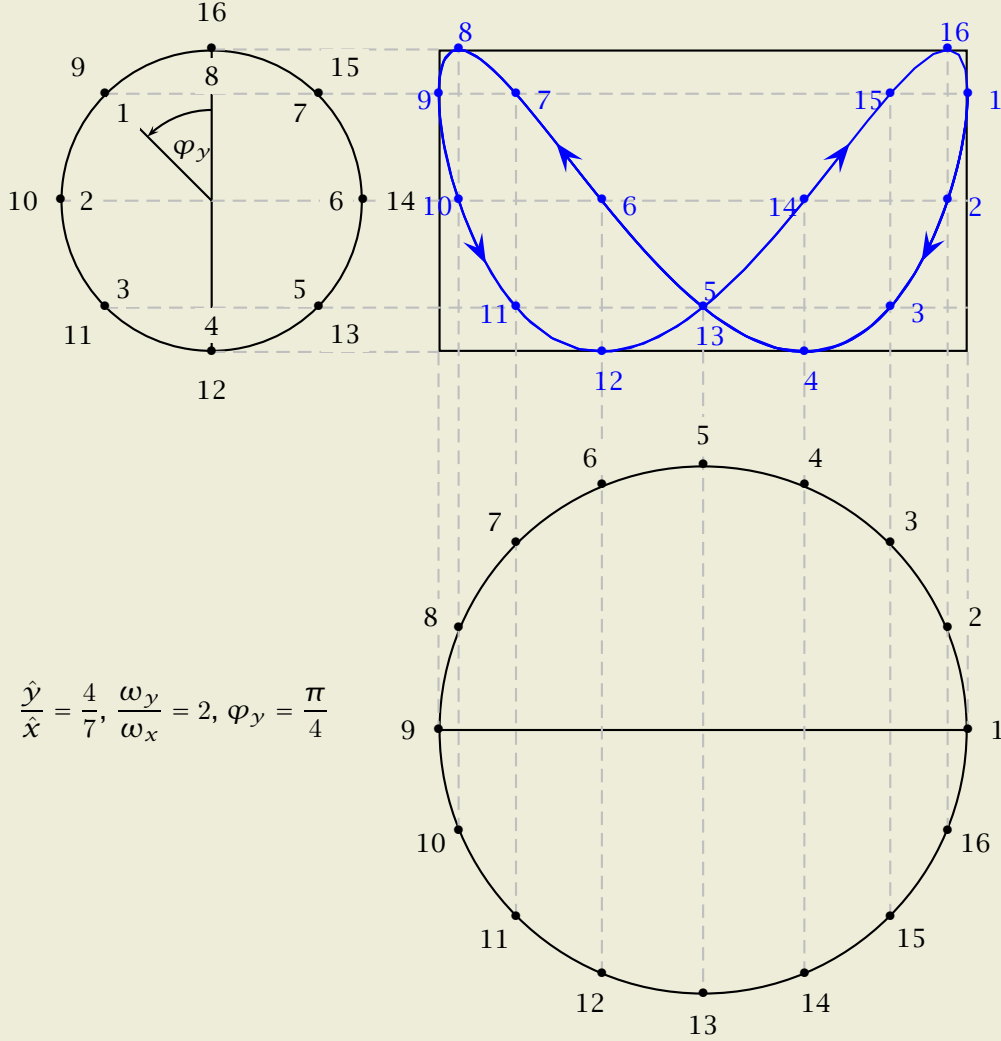


Figure 1: Example of the geometrical construction of a LISSAJOUS curve.

The orbits of the superposition of the perpendicular oscillations are called LISSAJOUS CURVES. These curves can be closed or open.

Let each oscillation be a simple harmonic motion represented by a sinusoidal wave; let $\hat{x}$ and $\hat{y}$ denote the amplitudes, ω_x and ω_y the angular frequencies (in radians per second), φ_x and φ_y the phases, and t the time.

We then have

$$x(t) = \hat{x} \cos(\omega_x t + \varphi_x), \quad (1)$$

$$y(t) = \hat{y} \cos(\omega_y t + \varphi_y). \quad (2)$$

Each of the harmonic oscillations can be drawn as a normal projection onto the coordinate axes. The projections are shown in this figure 1 above and give the projection point $P(x, y)$, which is part of the orbit.

Figure 1 also shows, that the movement of P in the (x, y) -plane is inside the range of a rectangle

$$-\hat{x} \leq x(t) \leq \hat{x}$$

and

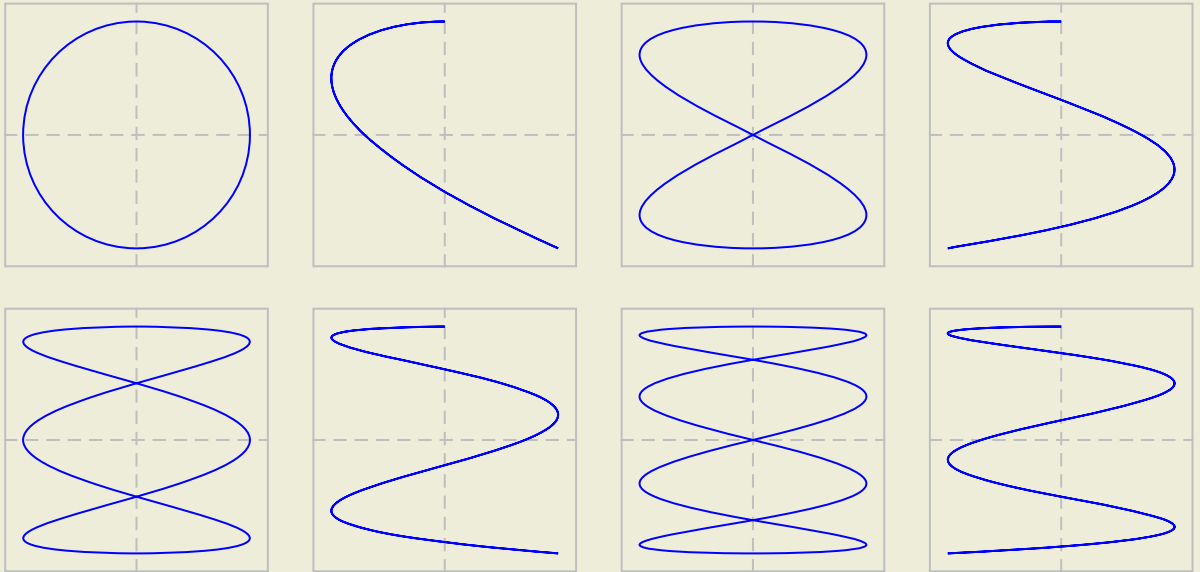
$$-\hat{y} \leq y(t) \leq \hat{y}.$$

1.1 Variation of angular frequency

Here an example with

$$\hat{x} = \hat{y} = 1.5 \text{ cm}, \omega_y = 1 \text{ s}^{-1}, \varphi_x = \frac{\pi}{2}, \varphi_y = 0$$

and the variation of ω_x .

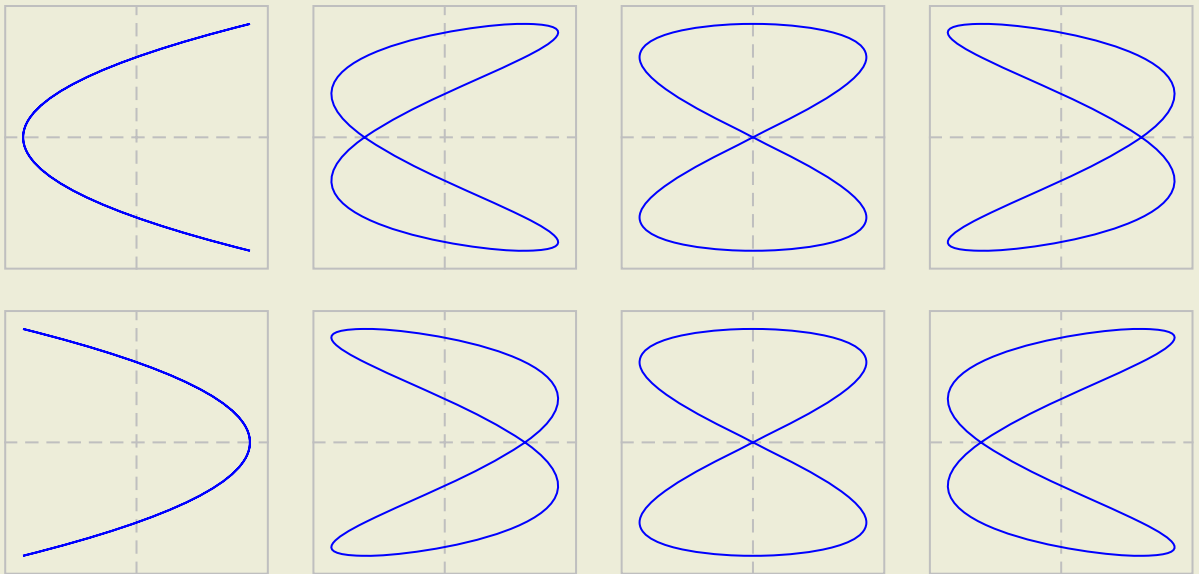


1.2 Variation of phase

Here an example with

$$\hat{x} = \hat{y} = 1.5 \text{ cm}, \omega_x = 2 \text{ s}^{-1}, \omega_y = 1 \text{ s}^{-1}, \varphi_y = 0$$

and the variation of φ_x .



1.3 PSTricks macro

The parameters of the following macro explains itself

```
\psLissajous[%  
hatx=3.5,  
haty=3.5,  
omegax=19,  
omegay=23,  
phix=45,  
phiy=0,  
linecolor=red,linewidth=0.5pt,plotpoints=5000]
```

and generates the following graphic

