The Gallery of Infinite Series

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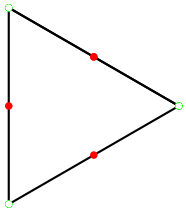
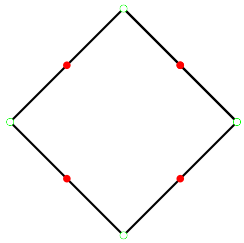
1 Real polygons

There are the polytopes $2\{\frac{p}{q}\}2$ (with p and q in $\mathbb{N}$) in the notation of Coxeter. Use the command:

```
\psset{unit=1.5cm}\Polygon[P=p,Q=q]
```

2	3	4
		
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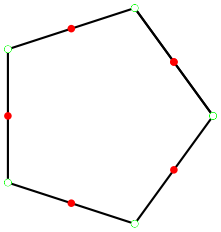	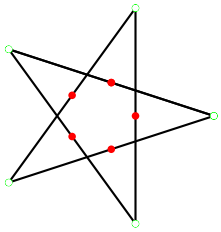	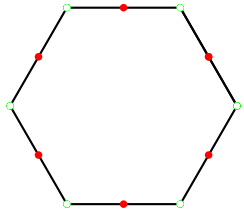
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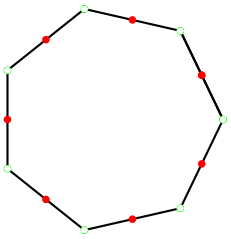	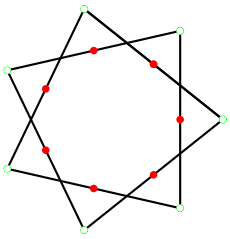	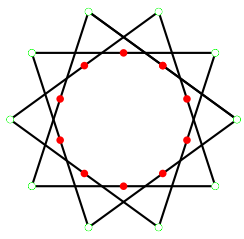

2 Simplices

There are the real polytopes $2\{3\}2\cdots 2\{3\}2$ in dimension n (tetrahedron, pentatope, sextatope etc...) in the notation of Coxeter. Use the command:

```
\psset{unit=1.5cm}\Simplex[dimension=n]
```

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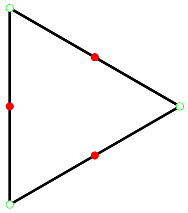	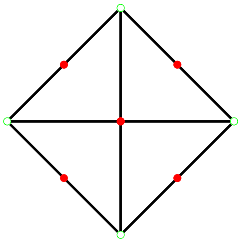	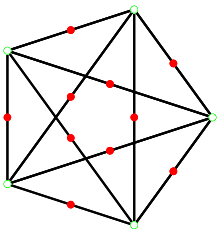
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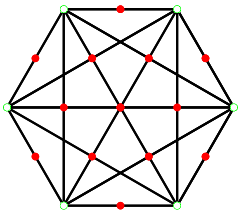
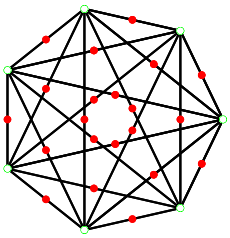
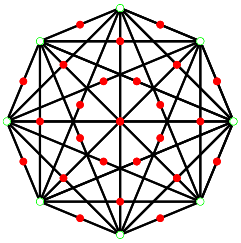
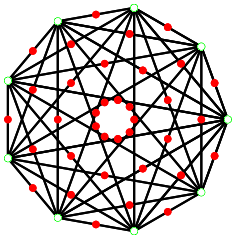
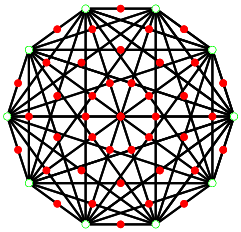
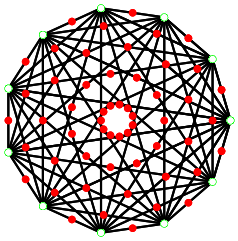
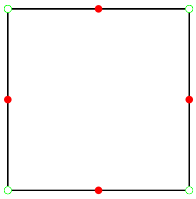
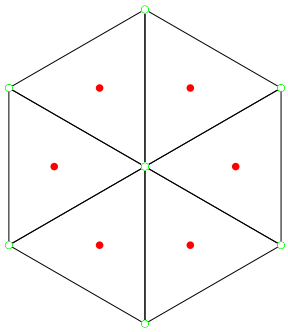
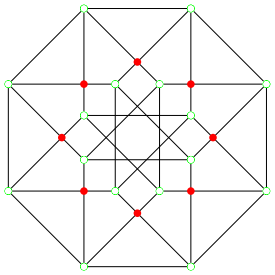
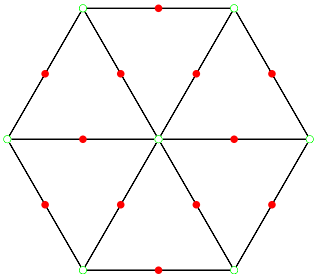
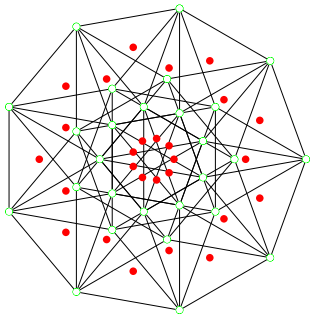
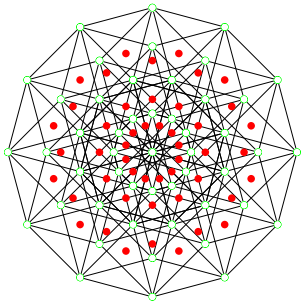
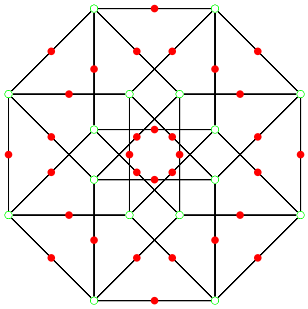
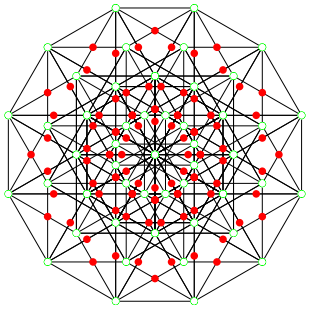
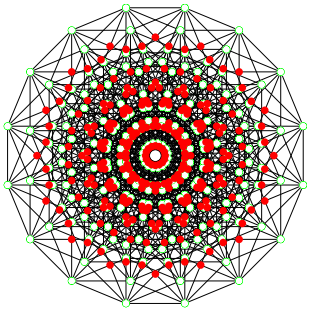
		
8	9	10
		

3 The infinite series γ_n^p

It is an infinite series of polytopes with two parameters p and n. The parameter n is the dimension of the polytope. In the notation of Coxeter, its name reads $p\{4\}2\{3\}\ldots\{3\}2$. In the case $p = 2$, we recovers the family of the hypercubes. Use the command:

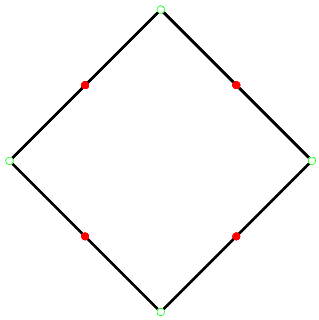
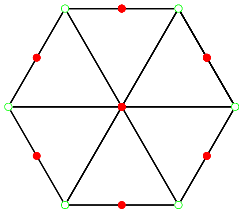
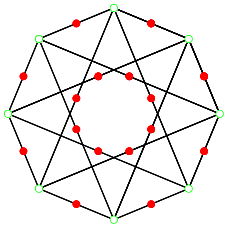
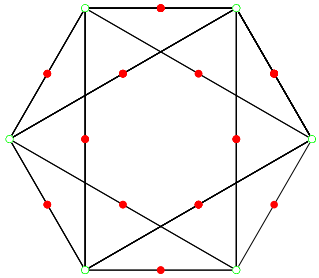
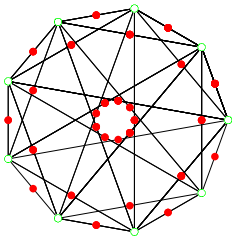
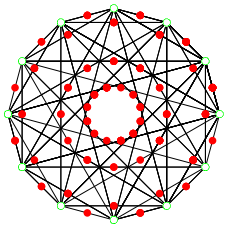
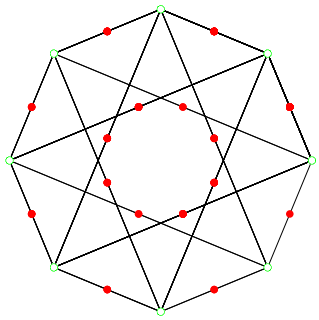
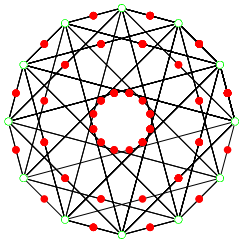
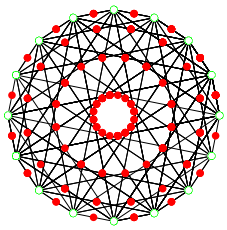
```
\gammapn[P=p,dimension=n]
```

γ_2^2	γ_2^3	γ_2^4
		
γ_3^2	γ_3^3	γ_3^4
		
γ_4^2	γ_4^3	γ_4^4
		

4 The infinite series β_n^p

It is an infinite series of polytopes with two parameters p and n reciprocals of γ_n^p. The parameter n is the dimension of the polytope. In the notation of Coxeter, its name reads $2\{3\}2\{3\}\ldots\{3\}2\{4\}p$. In the case $p = 2$, we recovers the family of the 2^n-topes which generalizes the tetrahedron for higher dimension. Use the command:

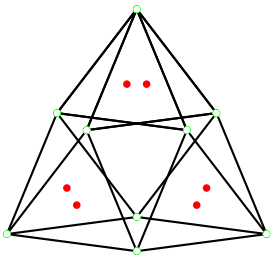
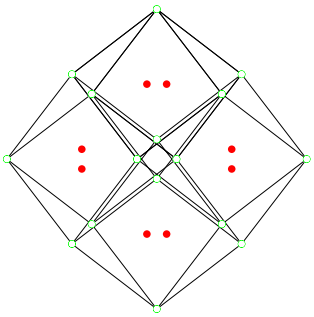
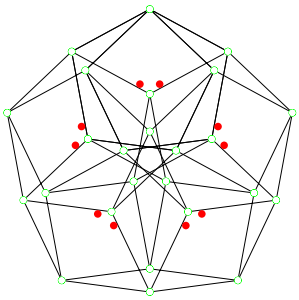
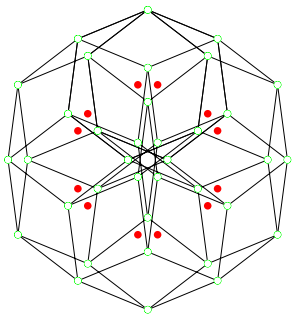
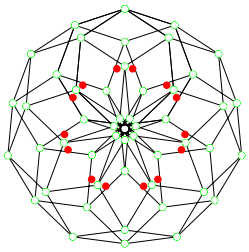
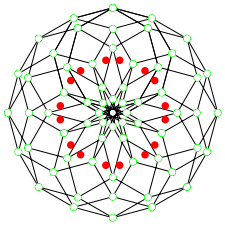
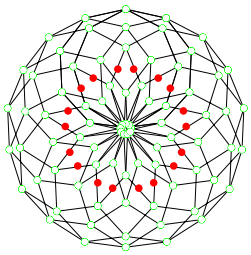
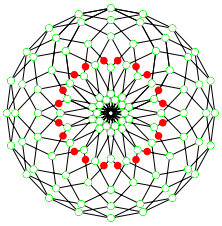
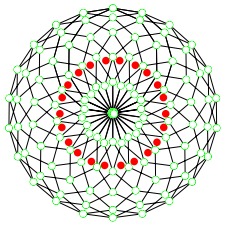
```
\betapn[P=p,dimension=n]
```

β_2^2	β_2^3	β_2^4
		
β_3^2	β_3^3	β_3^4
		
β_4^2	β_4^3	β_4^4
		

5 The infinite series γ_2^p

It is a special case of the series γ_n^p for $n = 2$. In this case, the polytopes are complex polygons. The projection used here is different than the projection used with `gammapn`. Use the command:

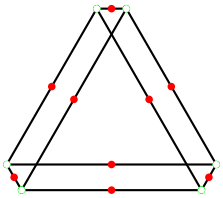
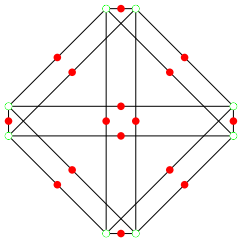
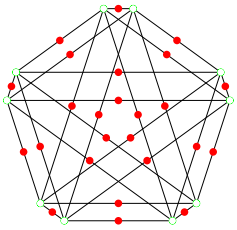
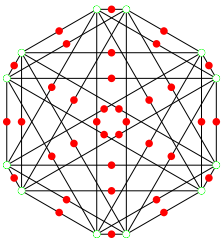
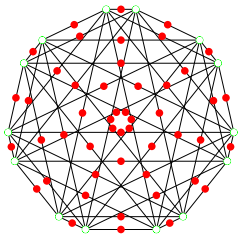
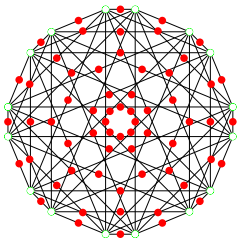
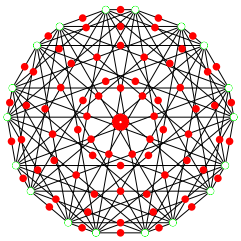
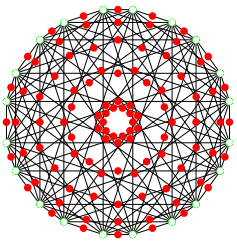
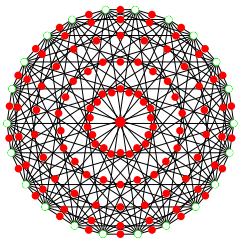
```
\gammaptwo[P=p]
```

γ_2^3	γ_2^4	γ_2^5
		
γ_2^6	γ_2^7	γ_2^8
		
γ_2^9	γ_2^{10}	γ_2^{11}
		

6 The infinite series β_2^p

It is a special case of the series β_n^p for $n = 2$. In this case, the polytopes are complex polygons. The projection used here is different than the projection used with `betapn`. Use the command:

```
\betaptwo[P=p]
```

β_2^3	β_2^4	β_2^5
		
β_2^6	β_2^7	β_2^8
		
β_2^9	β_2^{10}	β_2^{11}
		

References

[1] H. S. M. Coxeter, *Regular Complex Polytopes*, Second Edition, Cambridge University Press, 1991 .