The Library pst-coxeterp

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Abstract

We describe the `LaTex` library `pst-coxeterp` devoted to draw regular complex polytopes belonging in the infinite series.

1 Introduction

Inspired by the dissertation of G.C. Shephard [4], Coxeter toke twenty years to write his most famous book *Regular Complex Polytopes* [2]. But its interest for the polytope dates from the beginning of his career as shown his numerous publications on the subject (reader can refer to [1] or [3]). According to the preface of [2], the term of complex polytopes is due to D.M.Y. Sommerville [6]. A complex polytope may have more than two vertices on an edge (and in particular the polygons may have more than two edges at a vertice). It is a finite set of flags of subspaces in $\mathbb{C}^n$ with certain constraints which will be not explain here [1]. In fact, a complex polytope can be generated from one vertice by a finite number of pseudo-reflections. More precisely, as for the classical solids, it can be constructed from an arrangement of mirrors, considering a point in the intersection of all but one the mirrors and computing the orbit of this point by the pseudo-reflections generated by the mirrors. In the case of the real polytopes, one uses classical reflections which are involutions. It is not the case for general complex polytopes, since a reflection may include a component which is a rotation. The classification of the complex polytopes is due to G.C. Shephard [4] and is closely related to the classification of the complex unitary reflection groups [5]. This classification includes four infinite series of polytopes: the well-known real polygons (including the starry polygon) which have two parameters, the series of simplices (triangle, tetrahedron, pentatope, sextatope etc...) which have only one parameter, the dimension and to reciprocal series γ_n^p and β_n^p. The library described here is a `LaTex` package for drawing the polytopes of these infinite series.

2 Install `pst-coxeterp`

The package contains two files: A latex style file `pst-coxeterp.sty` which call the latex file `pst-coxeterp.tex` containing the description of the macros. The installation is very simple. It suffices to copy the files `pst-coxeterp.sty` and `pst-coxeterp.tex` in the appropriate directories.

Example 2.1 The file `pst-coxeterp.sty` may be copy in the directory
`c:/texmf/tex/latex/pst-coxeterp`,
the file `pst-coxeterp.tex` in
`c:/texmf/tex/generic/pst-coxeterp`

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[1]For a precise definition, see [2] Ch12

To use the package add the code

```
\usepackage{pst-coxeterp}
%
```

in the beginning of your LaTex-file.

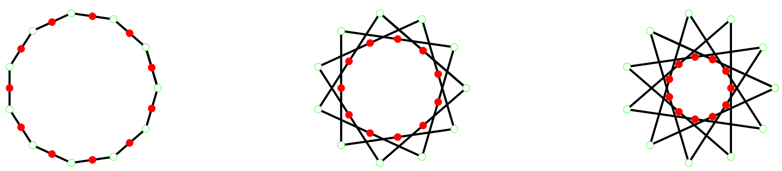
Example 2.2 `\documentclass[a4paper]{article}`

```
...
\usepackage{pst-coxeterp}
....
```

The library needs the packages `PSTrick` and `pst-xkey`.

3 The different families

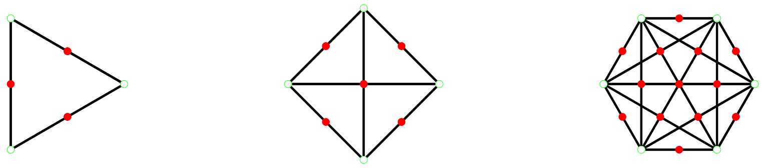
This library contains six macros for drawing polytopes belonging in a infinite series.
The first macro, `Polygon`, draws real (starry or not) polygon. The polygon is defined by two parameters `P` and `Q` which defines the angle $2\frac{Q}{P}\Pi$ between the segment from the center to the first vertices and the segment from the center to the second vertices. By default the value of `Q` is 1.



Example 3.1

```
\begin{pspicture}(-2,-2)(2,2)
\Polygon[P=11,Q=1] %
\end{pspicture}
\begin{pspicture}(-2,-2)(2,2)
\Polygon[P=11,Q=3]
\end{pspicture}
\begin{pspicture}(-2,-2)(2,2)
 \Polygon[P=11,Q=4]
\end{pspicture}
```

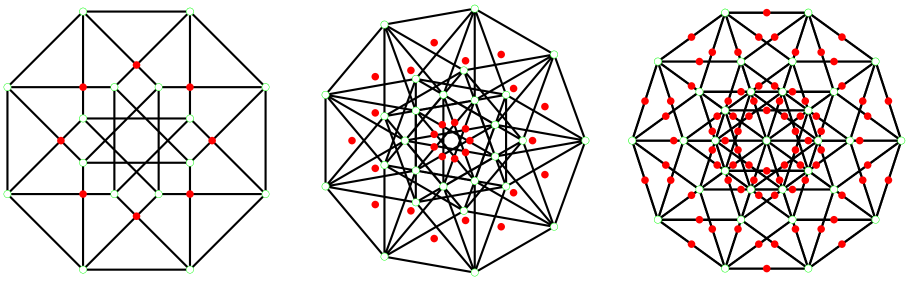
The macro `Simplex` draws simplices in dimension n. The simplices are the real polytopes whose automorphism groups are the symmetric groups. The dimension of the polytope can be chosen using the parameter `dimension`.



Example 3.2

```
\begin{pspicture}(-2,-2)(2,2)
\Simplex[dimension=2] %
\end{pspicture}
\begin{pspicture}(-2,-2)(2,2)
\Simplex[dimension=3]
\end{pspicture}
\begin{pspicture}(-2,-2)(2,2)
 \Simplex[dimension=5]
\end{pspicture}
```

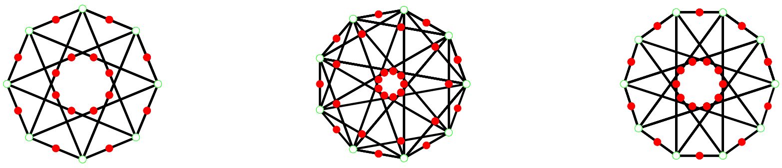
The polytopes γ_n^p forms a two parameters family which contains as special case the hypercubes. The parameter n is the dimension of the polytope and the parameter p is the number of vertices per edge. Use the macro `gammapn` and the parameters `dimension` and `P` to chose the characteristics of the polytope.



Example 3.3

```
\begin{pspicture}(-2,-2)(2,2)
\gammapn[dimension=2,P=4] %
\end{pspicture}
\begin{pspicture}(-2,-2)(2,2)
\gammapn[dimension=3,P=3,unit=0.7cm]
\end{pspicture}
\begin{pspicture}(-2,-2)(2,2)
 \gammapn[dimension=5,P=2,unit=0.55cm]
\end{pspicture}
```

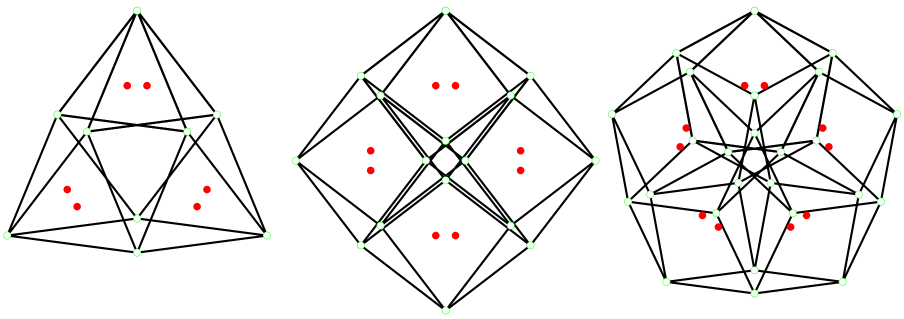
The polytopes β_n^p forms a two parameters family which contains as special case the hyperoctahedra. The parameter n is the dimension of the polytope and the parameter p is the number of cells of dimension $n-1$ containing a cell of dimension $n-2$. Use the macro `betapn` and the parameters `dimension` and `P` to chose the characteristics of the polytope.



Example 3.4

```
\begin{pspicture}(-2,-2)(2,2)
\betapn[dimension=2,P=4] %
\end{pspicture}
\begin{pspicture}(-2,-2)(2,2)
\betapn[dimension=3,P=3]
\end{pspicture}
\begin{pspicture}(-2,-2)(2,2)
 \betapn[dimension=5,P=2]
\end{pspicture}
```

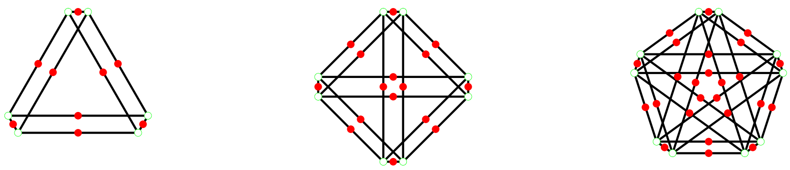
The macro `gammaptwo` draw the regular complex polytope γ_2^p which is a special case of γ_n^p for an other projection. Use the parameter `P` for setting the number of vertices by edge.



Example 3.5

```
\begin{pspicture}(-2,-2)(2,2)
\gammaptwo[P=3] %
\end{pspicture}
\begin{pspicture}(-2,-2)(2,2)
\gammaptwo[P=4]
\end{pspicture}
\begin{pspicture}(-2,-2)(2,2)
 \gammaptwo[P=5]
\end{pspicture}
```

The macro `betaptwo` draw the regular complex polytope β_2^p which is a special case of β_n^p for an other projection (the same than for `gammaptwo`). Use the parameter `P` for setting the number of vertices by edge.



Example 3.6

```
\begin{pspicture}(-2,-2)(2,2)
\betaptwo[P=3] %
\end{pspicture}
\begin{pspicture}(-2,-2)(2,2)
\betaptwo[P=4]
\end{pspicture}
\begin{pspicture}(-2,-2)(2,2)
 \betaptwo[P=5]
\end{pspicture}
```

4 Graphical parameters

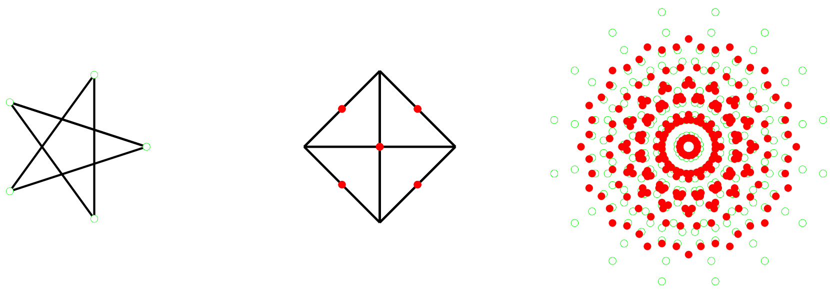
4.1 The components of a polytope

The library pst-coxeterrep.sty contains macros for drawing the vertices, the edges and the centers of the edges of polytopes of the infinite series of regular complex polytopes.

It is possible to choice which components of the polytope will be drawn. It suffices to use the boolean parameters drawedges, drawvertices and drawcenters.

By default the values of the parameters drawedges, drawvertices, drawcenters are set to true.

Example 4.1



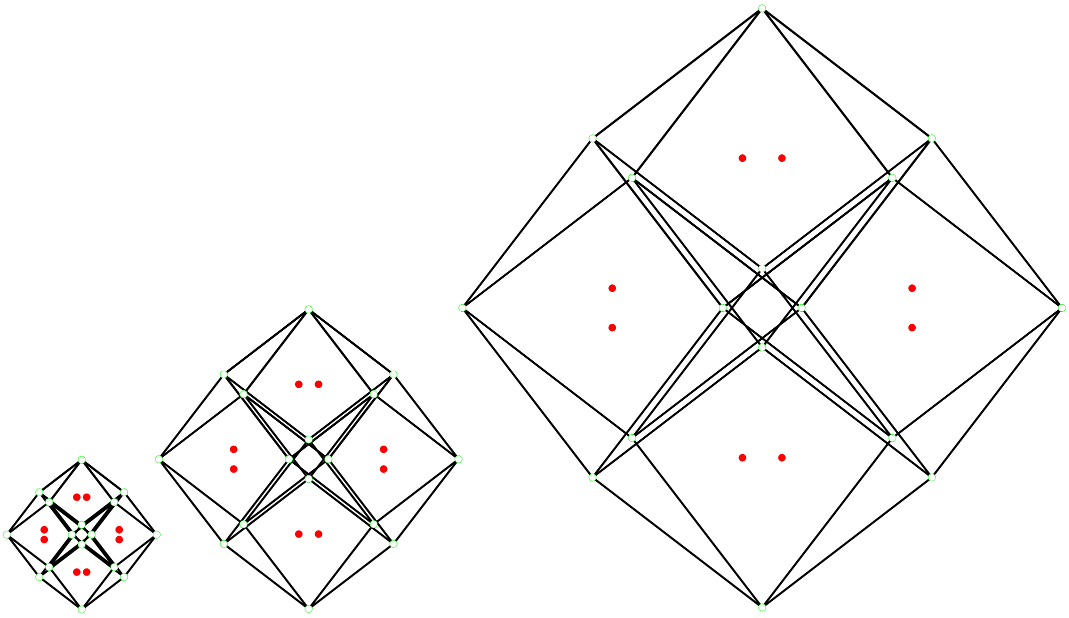
```
\begin{pspicture}(-2,-2)(2,2)
\Polygon[P=5,Q=2,drawcenters=false] %
\end{pspicture}
\begin{pspicture}(-2,-2)(2,2)
\Simplex[dimension=3,drawvertices=false] %
\end{pspicture}
\begin{pspicture}(-2,-2)(2,2)
```

```
\psset{unit=0.5}
 \gammapn[P=4,dimension=4,drawedges=false]
\end{pspicture}
```

5 Graphical properties

It is possible to change the graphical characteristics of a polytope.
The size of the polytope depends on the parameter `unit`.

Example 5.1



```
 \begin{pspicture}(-1,-1)(1,1)
\gammaptwo[P=4,unit=0.5cm] %
\end{pspicture}
 \begin{pspicture}(-2,-2)(2,2)
\gammaptwo[P=4,unit=1cm] %
\end{pspicture}
 \begin{pspicture}(-4,-4)(4,4)
\gammaptwo[P=4,unit=2cm] %
\end{pspicture}
```

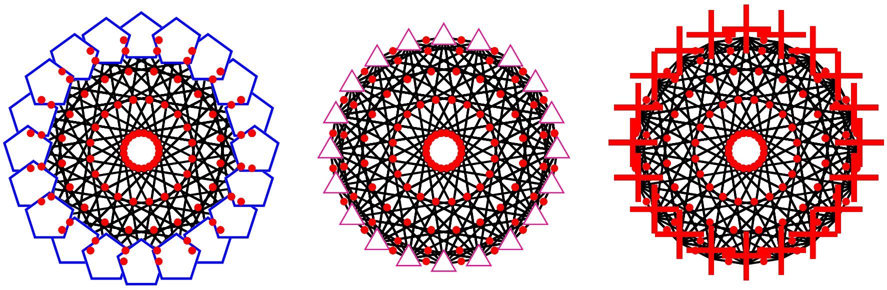
Classically, one can modify the color and the width of the edges using the parameter `linecolor` and `linewidth`.

Example 5.2

```
\begin{pspicture}(-2,-2)(2,2)
\psset{unit=0.8,linewidth=0.01,linecolor=red}
\betaptwo[P=5] %
\end{pspicture}
 \begin{pspicture}(-2,-2)(2,2)
\betaptwo[P=5] %
\end{pspicture}
```

The color, the style and the size of the vertices can be modify using the parameters `colorVertices`, `styleVertices` and `sizeVertices`. The style of the vertices can be chosen in the classical dot styles.

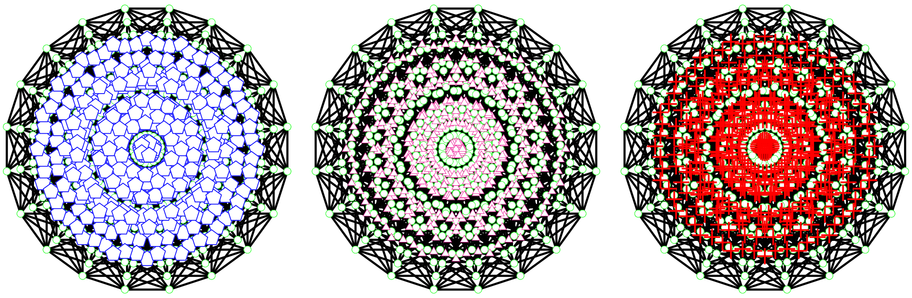
Example 5.3



```
\begin{pspicture}(-2,-2)(2,2)
\psset{unit=1.5cm,colorVertices=blue,styleVertices=pentagon,sizeVertices=0.2}
\betapn[P=5,dimension=4] %
\end{pspicture}
 \begin{pspicture}(-2,-2)(2,2)
\psset{unit=1.5cm,colorVertices=magenta,sizeVertices=0.1,styleVertices=triangle} %
\betapn[P=5,dimension=4]
\end{pspicture}
 \begin{pspicture}(-2,-2)(2,2)
\psset{unit=1.5cm,colorVertices=red,styleVertices=+,sizeVertices=0.2} %
\betapn[P=5,dimension=4]
\end{pspicture}
```

The color, the style and the size of the centers of the edges can be modify using the parameters `colorCenters`, `styleCenters` and `sizeCenters`.

Example 5.4



```
\begin{pspicture}(-2,-2)(2,2)
\psset{unit=0.5cm,colorCenters=blue,styleCenters=pentagon,sizeCenters=0.2} %
\gammapn[P=5,dimension=4] %
\end{pspicture}
 \begin{pspicture}(-2,-2)(2,2)
\psset{unit=0.5cm,colorCenters=magenta,sizeCenters=0.1,styleCenters=triangle} %
\gammapn[P=5,dimension=4] %
\end{pspicture}
 \begin{pspicture}(-2,-2)(2,2)
\psset{unit=0.5cm,colorCenters=red,styleCenters=+,sizeCenters=0.2} %
\gammapn[P=5,dimension=4] %
\end{pspicture}
```

References

[1] H. S. M. Coxeter, *Regular polytopes*, Third Edition, Dover Publication Inc., New-York, 1973.

[2] H. S. M. Coxeter, *Regular Complex Polytopes*, Second Edition, Cambridge University Press, 1991 .

[3] H.S.M. Coxeter, *Kaleidoscopes, selected writing of H.S.M. Coxeter by F.A. Sherk, P. McMullen, A.C. Thompson, A. Ivić Weiss*, Canadian Mathematical Society Series of Monographs and Advanced texts, Published in conjunction with the fiftieth anniversary of the canadian mathematical society, J. M. Borwein and P. B. Borwein Ed., A Wiley-Interscience publication, 1995.

[4] G.C. Shephard, *Regular Complex Polytopes*, Proceeding of the London Mathermatical Society (3), 2 82-97.

[5] G.C. Shephard and J.A. Todd, *Finite unitary reflection groups*, Canadian Journal of Mathematics 6, 274-304, 1954.

[6] M.Y. Sommerville, *Geometry of n dimension*, Methuen, Lodon, 1929.